10. $x^2 \cos x = x^2 - \frac{x^4}{2} + \frac{x^6}{4!} - \frac{x^8}{6!} + \cdots$

$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots$

24. $f(x) = 1 + \frac{x}{2!} + \frac{1 \cdot x^2}{3!} + \frac{x^3}{4!} + \cdots \frac{x^{10}}{11!}$

$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3 \cdots$

$f'(0) = \frac{1}{2}$ $\quad \frac{f''(0)}{2} = \frac{1}{3!} \cdot 2$ $\quad \frac{f^{10}(0)}{10!}x^{10}$

$\frac{f^{10}(0)}{\cancel{10!}} = \frac{1 \cdot 10!}{11!} = \frac{1}{11}$

$x f(x) = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \frac{x^n}{n!}$

c) $= e^x - 1$ $\quad (1+2+3+4) - 1$ $\quad$ n starts at 1

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$

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25. $\frac{e^x - 1}{x}$

a) $e^{\frac{x}{2}} = 1 + \frac{x}{2} + \frac{\left(\frac{x}{2}\right)^2}{2} + \frac{\left(\frac{x}{2}\right)^3}{3!}$

$= 1 + \frac{x}{2} + \frac{x^2}{2^2 \cdot 2} + \frac{x^3}{2^3 \cdot 3!} + \frac{x^4}{2^4 4!} \cdots$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$

b) $g(x) = \frac{e^x - 1}{x} = \frac{x}{x} + \frac{x^2}{x \cdot 2} + \frac{x^3}{x \cdot 3!} + \cdots$

$= 1 + \frac{x}{2} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots$

c) $g'(x) = \frac{1}{2} + \frac{2x}{3!} + \frac{3x^2}{4!} \cdots$ $\quad g'(1) = \frac{1}{2} + \frac{2}{3!} + \frac{3}{4!} \cdots$

$\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1$ $\quad g'(x) = \frac{xe^x - (e^x - 1) \cdot 1}{x^2}\Big|_{x=1} = 1$

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9.2b Taylor series

Maclaurin series - works for x close to 0

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f^{3}(0)}{3!}x^3 \ldots$$

Taylor series - works for x close to a

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{3}(a)}{3!}(x-a)^3 + \ldots$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!}(x-a)^n$$

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find the 1st 3 terms & general term for the Taylor series for e^x about $x=2$

$a=2$

$f(x)=e^x \quad f(a)=e^2$

$f'(x)=e^x \quad f'(a)=e^2$

$f''(x)=e^x \quad f''(a)=e^2$

$$e^x = e^2 + e^2(x-2) + \frac{e^2}{2}(x-2)^2 \ldots \frac{e^2}{n!}(x-2)^n$$

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1^{st} 4 terms of Taylor's Series for $f(x) = \sin x$

Centered on $x = \frac{\pi}{4}$

$f(x) = \sin x \quad f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$f'(x) = \cos x \quad f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$f''(x) = -\sin x \quad f''(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$f^3(x) = -\cos x \quad f'''(\frac{\pi}{4}) - -\frac{\sqrt{2}}{2}$

$$\sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2!\cdot 2}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{3!\cdot 2}\left(x - \frac{\pi}{4}\right)^3$$

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find mac series for $\frac{1+\cos(2x)}{2}$ 1^{st} 4 terms

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots$$

$$\cos(2x) = 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} \cdots$$

$$\frac{1+\cos(2x)}{2} = \frac{2}{2} - \frac{(2x)^2}{2\cdot 2} + \frac{(2x)^4}{2\cdot 4!} - \frac{(2x)^6}{2\cdot 6!} \cdots$$

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